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Gravitational Higgs Mechanism with a Topological Term

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Abstract

We investigate the Higgs mechanism for gravity, which has been recently put forward by 't Hooft, when the Polyakov-type action for scalar fields is added to the original action. We find that from the Polyakov-type action, it is very natural to derive an 'alternative' metric tensor composed of the scalar fields. The positivity condition on the determinant can be also derived easily by requiring that this term does not change the dynamics at all and becomes a topological number, that is, the wrapping number. It turns out that the gauge conditions adopted by 't Hooft are nothing but the restriction on a sector with unit wrapping number.

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1 Introduction

There have been recently some interesting works about the spontaneous symmetry breakdown (SSB) of general coordinate reparametrization invariance [1, 2, 3, 4]. The first motivation behind these works comes from brane world scenario where the presence of a brane breaks some of diffeomorphisms in the directions perpendicular to the brane spontaneously, so that we expect that there might naturally appear a gravitational Higgs mechanism in this context [1, 2].

The second motivation is that this phenomenon might play an important role in developing string theory approach to quantum chromodynamics (QCD) in future [4]. For instance, if we wish to apply a bosonic string theory to the gluonic sector in QCD, massless fields such as tachyonic scalar and spin 2 gravitons in string theory, must become massive or be removed somehow by some ingenious dynamical mechanism since such the fields do not exist in QCD.

As the final motivation, SSB of general coordinate reparametrization invariance might lead to some resolution for cosmological problems such as cosmological constant problem [3, 4].

Recently, 't Hooft has proposed an interesting Higgs mechanism for gravity where massless gravitons '*eat*' four real scalar fields, thereby becoming massive [4]. His motivation mainly lies in string theory approach to QCD and if the approach is effective, the massless gravitons must acquire a huge mass (perhaps, the Planck mass) and become unobservable at least in the low energy region. In this model, diffeomorphisms are broken by four real scalar fields spontanously such that vacuum expectation values (VEV's) of the scalar fields are chosen to the four space-time coordinates up to a proportional constant by gauge-fixing diffeomorphisms. Of course, the number of dynamical degrees of freedom is unchanged before and after SSB. Actually, before SSB of diffeomorphisms there are massless gravitons of two dynamical degrees of freedom and four real scalar fields whereas after SSB we have massive gravitons of five dynamical degrees of freedom and one real scalar field so that the number of dynamical degrees of freedom is equal to six both before and after SSB as desired.

A key observation in the 't Hooft model is that the scalar field appearing after SSB is a non-unitary propagating field so that in order to avoid violation of unitarity it must be removed from the physical Hibert space in terms of some procedure.[2] In fact, two methods were proposed at the classical level [4]. One method is to require that the energy-momentum tensor of the matter field does not couple to the usual metric tensor but the modified metric one in such a way that the non-unitary scalar field does not couple to the energy-momentum tensor directly. Another method is to eliminate the time-like component of the scalar fields by imposing a constraint on the scalar fields.[3] It is worthwhile to notice that in both the methods we have to introduce an '*alternative*' metric tensor constructed out of four real scalar

[2]More recently, a model of gravitational Higgs mechanism without the non-unitary propagating scalar field was constructed in a conformally flat expanding background in Ref. [5].

[3]In order to match the Minkowskian signature of the background, one of the four scalar fields must take a negative signature. Or equivalently, in the Euclidean signature after the Wick rotation, one of the four scalar fields must have an imaginary vacuum expectation value.

fields, but its derivation from the first principle is lacking.[4]

In this short article, we investigate a possibility of having a topological term in the 't Hooft theory. Topology has thus far played a central role in quantum field theories so it is worth pursuing such a possibility. To do that, we incorporate the Polyakov-type action to the 't Hooft's starting action and explore how this term behaves when we require that this additional term should not change the dynamics completely. Similar but different approaches have been already taken into consideration in Ref. [6].

Let us start with the following Euclidean action in four space-time dimensions. This action differs from the 't Hooft action [4] only by the last term S_P:

$$S = S_{EH} + S_\Lambda + S_\phi + S_M + S_P, \tag{1}$$

where each term takes the following form:

$$\begin{aligned}
S_{EH} &= \frac{1}{16\pi G}\int d^4x\sqrt{g}R, \\
S_\Lambda &= -\frac{\Lambda}{8\pi G}\int d^4x\sqrt{g}, \\
S_\phi &= -\frac{1}{2}\int d^4x\sqrt{g}g^{\mu\nu}\partial_\mu\phi^a\partial_\nu\phi^a, \\
S_M &= \int d^4x\mathcal{L}_{matters}, \\
S_P &= -\frac{T}{2}\int d^4x\sqrt{g_\phi}g_\phi^{\mu\nu}\partial_\mu\phi^a\partial_\nu\phi^a + \Lambda_P\int d^4x\sqrt{g_\phi}.
\end{aligned} \tag{2}$$

Here the fourth term S_M describes an action for a general matter field but ϕ^a.

Now let us take a variation with respect to $g_\phi^{\mu\nu}$, which gives us the famous equations of motion in string (or brane) theory:

$$\begin{aligned}
0 &= T^\phi_{\mu\nu} \\
&= \partial_\mu\phi^a\partial_\nu\phi^a - \frac{1}{2}g^\phi_{\mu\nu}g_\phi^{\gamma\delta}\partial_\gamma\phi^a\partial_\delta\phi^a + \frac{\Lambda_P}{T}g^\phi_{\mu\nu}.
\end{aligned} \tag{3}$$

It is easy to solve the equations whose result reads

$$g^\phi_{\mu\nu} = \frac{T}{\Lambda_P}\partial_\mu\phi^a\partial_\nu\phi^a. \tag{4}$$

[4] Here we would like to emphasize that there is no rule that we have only unique metric tensor in our world. For instance, there might be a possibility such that we have two distinct metric tensors in our world, one of which controls the macroscopic, cosmological region while the other metric tensor does the microscopic, elementary particles' region. Then, a real problem is to understand the relationship between two metric tensors in the intermediate region.

For simplicity, we henceforth assume $T = \Lambda_P$.[5] In this way, we arrive at the expression of an '*alternative*' metric tensor constructed out of four real scalar fields

$$g^{\phi}_{\mu\nu} = \partial_\mu \phi^a \partial_\nu \phi^a. \tag{5}$$

Note that this relation was assumed in an ad hoc manner in Ref. [4] whereas it is now derived from the action principle.

Next, let us rewrite the Polyakov-type action to the Nambu-Goto-type one by substituting the relation (5) into S_P:

$$\begin{aligned} S_P &= -\Lambda_P \int d^4x \sqrt{g_\phi} \\ &= -\Lambda_P \int d^4x \sqrt{\det_{\mu,\nu} \partial_\mu \phi^a \partial_\nu \phi^a}, \end{aligned} \tag{6}$$

where we have appended the indices μ, ν to the determinant in order to emphasize that this is the determinant for a matrix with row index μ and column one ν. Now one finds that one can recast this equation further when the number of space-time dimensions is equal to that of scalar fields, which just corresponds to the situation at hand:

$$S_P = -\Lambda_P \int d^4x |\det_{\mu,a} \partial_\mu \phi^a|, \tag{7}$$

where the determinant with respect to μ and ν is replaced with the one with respect to μ and a. Moreover, we should take the absolute value of the determinant.[6]

Note that at this stage the term S_P is almost topological in the sense that at least locally one can eliminate all the dynamical degrees of freedom associated with four scalar fields by using diffeomorphisms in four space-time dimensions. Nevertheless this term is not completely topological in that there is an ambiguity coming from the absolute value. Then, we require that this additional term S_P should not change the dynamics of the original 't Hooft theory both locally and globally. In order to do so, we must pick up either a positive sign or a negative one. From now on, we shall confine ourselves to a positive sign:

$$\det_{\mu,a} \partial_\mu \phi^a > 0. \tag{8}$$

Then, we have a completely topological term for S_P:

$$\begin{aligned} S_P &= -\Lambda_P \int d^4x \det_{\mu,a} \partial_\mu \phi^a \\ &= -\Lambda_P \int d^4x \varepsilon^{\mu\nu\rho\sigma} \partial_\mu \phi^1 \partial_\nu \phi^2 \partial_\rho \phi^3 \partial_\sigma \phi^4, \end{aligned} \tag{9}$$

[5] A different value from this value is in essence equivalent, if we make a suitable rescaling of $g^{\phi}_{\mu\nu}$. Then, this results in a multicative coefficient in the relation between $g^{\phi}_{\mu\nu}$ and $\partial_\mu \phi^a \partial_\nu \phi^a$.

[6] Some references miss putting the absolute value.

which is nothing but the wrapping number $\Pi_3(S^3) = Z$ up to an overall constant.[7]

Here let us consider the gauge conditions for diffeomorphisms in [4], which are given by

$$\phi^\mu = m x^\mu. \tag{10}$$

It was pointed out in [4] that there are ambiguities in the gauge-fixing conditions (10), for which the condition (8) is imposed by hand. Recall that this condition also emerges in the cure of the indefinite metric problem. From the present point of view, the condition (8) appears in order to make the Polyakov-type term completely topological. Then, what is the mathematical meaning of the gauge-fixing conditions (10)? As is easily shown, with the gauge-fixing conditions (10) the wrapping number takes one (if we take $m = 1$), so the gauge conditions (10) mean that we are in a topological sector with unit wrapping number. To put differently, together with the condition (8) and the gauge-fixing conditions (10), the 't Hooft theory is uniquely defined in the Hilbert space with unit wrapping number. In this context, one can conjecture that the model might be generalized to the more general Hilbert space where the wrapping number takes a more general value, by choosing different conditions from (8) and (10).

In conclusion, in this article, we have investigated a possibility of having a topological term within the framework of the Higgs mechanism for gravity. The results obtained so far make clear that such a topological approach is very useful even in this theory as in the conventional quantum field theories. We have derived an '*alternative*' metric tensor constructed out of four real scalar fields by starting with the action. Furthermore, we have shed a new light on the interpretation for both the positivity of the determinant and the gauge-fixing conditions.

Finally, we wish to comment on two methods of removing a non-unitary propagating scalar field, which were presented in [4]. In these methods, we have to impose an additional constraint by hand. Although such a procedure is reasonable at the classical level, it might lead to some inconsistency at the quantum level. Maybe, a more plausible method is to introduce such a constraint in the theory as the gauge-fixing condition for an extra local symmetry. As one of such the methods, in future we would like to take account of a theory where there is the gauge field A_μ and one real scalar since the total number of dynamical degrees of freedom is three, which is the minimum number for the gravitational Higgs mechanism in four dimensions [7]. In this method, the time-like component A^0 in the gauge field could be removed through the usual gauge invariance.

Note added

During preparation of this article, a preprint [5] has appeared where an idea of introducing the gauge field and one real scalar was commented. Moreover, more recently, a related preprint [8] has been also put on the archive.[8]

[7] In case of the 't Hooft theory, the four-dimensional space-time has an asymptotically flat boundary and after topological compactification with one point added, the boundary becomes S^3.

[8] We wish to thank D. Sorokin for informing me of the preprint [8].

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